

BEHAVIOUR OF OPTIMAL RESERVE INVENTORY BETWEEN TWO MACHINES IN SERIES BASED ON THE REPAIR TIME DISTRIBUTION

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Abstract

Determining the optimal reserve inventory between machines in series is a special type of inventory problem in the inventory control theory. The models for optimal reserve inventory between two machines have been discussed by many researchers with the assumption that the repair time of M_1 undergoes parameteric change and change of distribution properties. In this paper, a study on the behaviour of optimal reserve inventory between two machines in series is discussed. Especially the behaviour of optimal reserve inventory is analysed for the changes in the holding costs and ideal time costs, whenever the repair time of machine M_1 undergoes the Setting Clock Back to Zero and Change of distribution property. Also the optimal reserve inventory values are compared under the above two said properties.

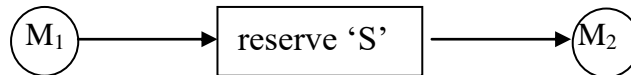
Keywords: Series system, Semi-finished products, Repair time, Breakdown period, Truncation point.

Introduction

Determination of optimal inventory under different real life situations is carried out by depicting the real life situation in terms of a mathematical model. When working systems are considered, avoiding the breakdown of the system is possible by keeping a reserve inventory between machines especially when the machines are in series.

A system in which there are two machines M_1 and M_2 in series is considered. The output of machine M_1 is the input for M_2 . The break down of M_1 will lead to the idle time

of M_2 . The idle time of M_2 will lead to reduce the productivity and to provide loss. Hence the idle time (or) stopping of M_2 is to be avoided. Therefore, a reserve inventory of size 'S' is to be maintained in between machines M_1 and M_2 . The system can be visualized as under



The output from M_2 is the finished product. If the reserve inventory 'S' is in excess there is inventory holding cost. If the reserve inventory is very small it may result in the idle time of M_2 , due to the fact that reserve may get exhausted by the time the machine M_1 is repaired and starts production. Hence the problem is to obtain the optimal reserve inventory \hat{S} between M_1 and M_2 . Hanssman (1962) has initiated the problem of finding an optimal reserve inventory between two machines in series when the output of M_1 is input for M_2 during the working and downstate of M_1 under deterministic demand for the input of M_2 . Suresh kumar (2006) has used exponential and Erlang 2 distributions as the threshold levels before and after the change point, a model is obtained for the expected time to the breaks down of the system and its variance through the technique of the shock model and cumulative damage process approach. Sehk Uduman et al (2007) have studied a model and derived the optimal reserve inventory between two machines in series using the concept of order statistics. Sachithanatham et al (2007) have obtained the optimal size of the reserve inventory under the assumption that the repair time distribution satisfies the SCBZ property. Ramathilagam et al (2014) have assumed that the repair time of M_1 is a random variable follows exponential distribution undergoes a parametric change after the truncation point and which is taken to be a random variable. The optimum reserve inventory is obtained under the assumption that the repair time having the SCBZ property. Sachithanatham and Jagatheesan (2017) have designed a model and the repair times of machine is assumed to be a random variable and it follows exponential distribution which satisfies the change of distribution property.

Also, the truncation point of the repair time distribution is itself a random variable and it follows mixed exponential distribution.

Description of the Inventory systems

The measures of inventory problems have been studied by many researchers based on various costs and other allied concepts. The researchers on inventory systems have studied the optimization of inventory levels, optimum cost, optimum order quantity and so on. The systems dealt with different statistical distributions and restrictions. It is very difficult to compare all these models. For this study, only three models are selected to analyse and compare the nature of inventory models. The models are stated as “The reserve inventory between two machines in series when the repair time of first machine falls before and after truncation point” and respectively it

- 1) follows Exponential and Erlang distributions
- 2) satisfies Switching Clock Back to Zero property
- 3) follows the density function proportional to random variable and uniform distribution.

Model I:

Srinivasan et al (2012) have constructed an inventory model with two machines in series. The major condition of this model is that the output of machine M_1 is the input of Machine M_2 . Suppose the machine M_1 breaks down and unable to supply the semi-finished products to machine M_2 , then the machine M_2 becomes idle. The idle time of M_2 leads the loss in the production. Here for avoiding the idle time, the reserve inventory is placed by semi-finished products in between machines M_1 and M_2 . In this juncture, there are mainly two costs such as holding and shortage costs arise.

Assumptions

- There are two machines in series and the output of machine M_1 is the input for the M_2 .
- The consumption rate of M_2 is a constant denoted by r .
- The repair time of M_1 is a random variable and it undergoes a change of distribution after a truncation point over the time axis. The repair time of M_1 follows exponential distribution with parameter θ_1 before the truncation point and Erlang 2 distribution after the truncation point.

Notations

- τ : a random variable denoting the repair time of M_1
- $\tau \sim$ exponential with parameter θ_1 if $\tau \leq t$
- $\tau \sim$ Erlang 2 with parameter θ_2 if $\tau > t$
- t : truncation point or change point
- $g(\tau)$: the pdf of τ
- h : the inventory holding cost
- d : the idle time cost of M_2
- $\frac{1}{\mu}$: average number of breakdowns per unit of time of M_1
- S : size of the reserve inventory between M_1 and M_2
- \hat{S} : the optimum size of reserve inventory between M_1 and M_2

It is assumed that the repair time of M_1 is a random variable τ and it follows exponential distribution with parameter θ_1 before the truncation point and Erlang type 2 distribution with parameter θ_2 after the truncation point. Similarly, the consumption rate of M_2 is a constant denoted by 'r'. The pdfs of the random variable τ are written as

$$\begin{aligned} g_1(\tau) &= \theta e^{-\theta_1 \tau} P[\tau < t] \\ &= \theta_1 e^{-\tau(\theta_1 + \lambda)} \end{aligned} \quad \dots(1)$$

and

$$g_2(\tau) = \theta_2^2(\tau - t)e^{-\theta_2(\tau-t)}e^{-\theta_1 t}P[\tau > t]$$

$$= \frac{\lambda\theta_2^2}{(\theta_1 - \theta_2 + \lambda)^2} [e^{-(\theta_1 + \lambda)\tau}e^{-\theta_2 \tau} + (\theta_1 - \theta_2 + \lambda)\tau e^{-\theta_2 \tau}] \dots(2)$$

In order to find out the optimum level of inventory between two machines, first of all, construct cost equations based on the probability density functions given in (1) and (2) and other key parameters including holding and shortage costs. The cost equations are formed in two cases such as considering the distribution of the random variable lies before and after transaction point of repair time.

Case (1): Before truncation point

Assuming the truncation point t and for the case $\frac{s}{r} < t$, the total expected cost based on the assumptions stated above, is given by

$$E(c) = hr \int_0^{s/r} \left(\frac{s}{r} - \tau\right) g_1(\tau) d\tau + \frac{d}{\mu} \int_{s/r}^t \left(\tau - \frac{s}{r}\right) g_1(\tau) d\tau + \frac{d}{\mu} \int_t^\infty \left(\tau - \frac{s}{r}\right) g_2(\tau) d\tau \dots(3)$$

Substitute the expressions for $g_1(\tau)$ and $g_2(\tau)$ in the equation (3). Using Leibnitz rule of differentiation of integral and get the required solution.

$$\hat{S} = \frac{r}{(\theta_1 + \lambda)} \left[\log_e \left\{ \frac{r\mu(\theta_1 + \lambda)}{\theta_1(d + \mu h)} \right\} \left[\frac{h\theta_1}{r(\theta_1 + \lambda)} \left(h + \frac{d}{\mu} e^{-t(\theta_1 + \lambda)} \right) + \right. \right.$$

$$\left. \left. + \frac{d}{r\mu(\theta_1 + \lambda)(\theta_1 - \theta_2 + \lambda)^2} \{ (r\theta_2 - \lambda(1 + \theta_2)(\theta_1 - \theta_2 - \lambda))(\theta_1 + \lambda) e^{-t\theta_2} - \lambda\theta_2^2 e^{-t(\theta_1 + \lambda)} \} \right] \right] \dots(4)$$

Case (2) : After truncation point

Consider the truncation point t and its the total expected cost relating to the inequality $t \leq \frac{s}{r}$ is given by

$$E(c) = hr \int_0^t \left(\frac{s}{r} - \tau\right) g_1(\tau) d\tau + hr \int_t^{s/r} \left(\frac{s}{r} - \tau\right) g_2(\tau) d\tau + \frac{d}{\mu} \int_{s/r}^{\infty} \left(\tau - \frac{s}{r}\right) g_2(\tau) d\tau \dots(5)$$

Substituting the expression for $g_1(\tau)$ and $g_2(\tau)$ given in the equations (1) and (2) in the equation (3) and using Leibnitz rule of differentiation of integral. The required result involving holding cost, shortage cost and inventory level is obtained as

$$\begin{aligned} \frac{dE(c)}{ds} &= \frac{h\theta_1}{r(\theta_1+\lambda)} [1 - e^{-t(\theta_1+\lambda)}] + \frac{h\lambda\theta_2^2}{r(\theta_1-\theta_2+\lambda)^2} \left[\frac{1}{(\theta_1+\lambda)} \left\{ e^{-t(\theta_1+\lambda)} - e^{-\frac{s}{r}(\theta_1+\lambda)} \right\} \right. \\ &+ \frac{1}{\theta} \left\{ e^{-\theta_2 \frac{s}{r}} - e^{-t\theta_2} \right\} + \frac{(\theta_1-\theta_2+\lambda)}{\theta_2^2} \left\{ (1+t\theta_2)e^{-t\theta_2} - \left(1 + \frac{s\theta_2}{r}\right) e^{-\frac{s}{r}\theta_2} \right\} \\ &+ \left. \frac{d\lambda\theta_2^2}{r\mu(\theta_1-\theta_2+\lambda)^2} \left[\frac{e^{-\frac{s}{r}(\theta_1+\lambda)}}{(\theta_1+\lambda)} + \frac{e^{-\frac{s}{r}\theta_2}}{\theta_2} - \frac{(\theta_1-\theta_2+\lambda)}{r\theta_2^2} \{r + s\theta_2\} e^{-\frac{s}{r}\theta_2} \right] \right] = 0 \dots(6) \end{aligned}$$

Model II

Sachithanantham et al (2006) have designed a model with two machines in series. The repair time of machine M_1 is a random variable t with p.d.f. $g(t)$ which satisfies the Setting the Clock Back to Zero (SCBZ) property. Raja Rao and Talwalker (1990) have introduced this SCBZ property. According to this property, the probability distribution of the random variable t has a parametric change from θ to θ^* . It may be stated that

$$f(x, \theta) \rightarrow f(x, \theta^*); \quad x > x_0$$

A condition which ensures the existence of the SCBZ property for any distribution is that

$$\frac{S(x+x_0, \theta^*)}{S(x_0, \theta)} = S(x, \theta^*)$$

where $S(x_0, \theta)$ is the survivor function defined as $1-F(x_0) = P [X \geq x_0]$.

Notations

- h : inventory holding cost / unit / unit time
d : idle time cost due to M_2 / unit time
r : constant rate of consumption M_2 / unit time
t : a continuous r.v. denoting the repair time of M_1 and with $g(\cdot)$ as pdf and $G(\cdot)$ as c.d.f.
U : a random variable denoting the inter arrival times between successive breakdowns of M_1 which is taken as exponential with parameter μ .
S : Reserve inventory between M_1 and M_2

It is noted that the random variable t follows $g(t, \theta)$, if $t < x_0$ and it follows $g(t, \theta^*)$, if $t \geq x_0$ where x_0 is the truncation point. Here, also costs are analysed in two different cases.

Assuming the pdf of t with parameter θ is given by

$$g(t, \theta) = \frac{1}{\theta} e^{-t/\theta}, \quad 0 \leq t \leq x_0 \quad \dots (7)$$

$$g(t, \theta^*) = \frac{1}{\theta^*} e^{-t/\theta^*}, e^{x_0(\frac{1}{\theta^*} - \frac{1}{\theta})} \quad t > x_0 \quad \dots (8)$$

The probability density functions of (7) and (8) are defined as $G(x_0, \theta)$ and $G(x_0, \theta^*)$ respectively.

Case (1) : Before Truncation Point

Suppose $\frac{S_0}{r}$ lies before x_0 , $\frac{S_0}{r} \leq x_0$, where S_0 is the initial reserve inventory level, then the total expected cost is framed as

$$E(c) = h S_0 + \frac{d}{\mu} \int_{S_0/r}^{x_0} \left(t - \frac{S_0}{r}\right) g(t, \theta) dt + \frac{d}{\mu} \int_{x_0}^{\infty} \left(t - \frac{S_0}{r}\right) g(t, \theta^*) dt \quad \dots (9)$$

Differentiating the equation (9) with respect to S_0 and equating to zero.

$$\frac{dE(c)}{dS_o} = \Rightarrow$$

$$h + \frac{d}{\mu} \left[\frac{d}{dS_o} \left\{ \int_{\frac{S_o}{r}}^{x_o} \left(t - \frac{S_o}{r} \right) g(t, \theta) dt + \int_{x_o}^{\infty} \left(t - \frac{S_o}{r} \right) g(t, \theta^*) dt \right\} \right] = 0 \quad \dots(10)$$

By applying Leibnitz rule of differentiation of integral and get the required solution.

$$G(x_{0,\theta}) - G\left(\frac{S_o}{r}, \theta\right) + \bar{G}(x_o, \theta^*) = \frac{h\mu r}{d} \quad \dots(11)$$

Now apply the cumulative distribution functions of the pdfs given in (7) and (8) in the equation (11) and get,

$$\widehat{S}_o = r \theta \log(d/h\mu r) \quad \dots(12)$$

Case (2): After truncation point

Secondly consider the ratio $\frac{S_o}{r}$ falls after the truncation point x_o . Based on the condition $x_o < \frac{S_o}{r}$, the total expected cost is designed as

$$E(c) = h S_o + \frac{d}{\mu} \int_{\frac{S_o}{r}}^{\infty} \left(t - \frac{S_o}{r} \right) g(t, \theta^*) dt \quad \dots(13)$$

As usual differentiate (13) with respect to S_o and equating to zero

$$\frac{dE(c)}{dS_o} = \Rightarrow$$

$$h - \frac{d}{\mu r} \int_{S_o/r}^{\infty} g(t, \theta^*) dt = 0$$

$$\Rightarrow \int_{S_o/r}^{\infty} g(t, \theta^*) dt = \frac{h\mu r}{d} \quad \dots(14)$$

Substitute the expression (8) in (14) which gives

$$\widehat{S}_o = r \theta^* \left\{ \log \left(\frac{d}{h\mu r} \right) + x_o \left(\frac{1}{\theta^*} - \frac{1}{\theta} \right) \right\} \quad \dots(15)$$

Model III

Nandakumar and Srinivasan (2015) have discussed a problem in which a manufacturing process is undergoing two processes through the machines M_1 and M_2 in series. Breakdown time of M_1 , the fluctuations of inventory sizes between two machines

and idle time of M_2 are considered to analyse the model. Due to the fluctuations of the inventory sizes, the holding cost and shortage cost are measured.

Assumptions

- When machine M_1 goes to downstate, the supply to the machine M_2 is from the reserve inventory.
- The repair time of M_1 is a random variable
- Within a short duration the reserve inventory S is replenished after each breakdown of M_1

Notations

X : continuous r.v denotes the breakdown time of M_1 whose p.d.f. is $f(x)$ and c.d.f is $F(x)$.

τ : chane point or truncation point is a random variable denoting the duration of breakdown of M_1 and its pdf is $g(\cdot)$ with cdf $G(\cdot)$

S : reserve inventory

\mathcal{S} : optimum reserve inventory

μ : mean time interval between breakdowns of machine M_1

d : cost per unit time of idle time of machine M_2

h : cost per unit time of holding one unit of reserve inventory

r : consumption rate per unit time of machine M_2

Let X be a breakedown time random variable and follows Exponential distribution with parameter θ before τ and Gamma 2distribution with parameter k after τ

$$g(x) = \begin{cases} \theta e^{-\theta x} & , 0 < x \leq \tau \\ \frac{e^{-\theta \tau} 2^k (x-\tau)^{k-1} e^{-2(x-\tau)}}{\Gamma k} & , x > \tau \end{cases} \quad \dots(16)$$

The mean of the model is stated as $E(x) = \frac{1}{\theta} + e^{-\theta \tau} \left(\frac{k}{2} - \frac{1}{\theta} \right) \quad \dots(17)$

In this stage, we recall the result designed by Hanssman (1962) for solving this problem.

$$G\left(\frac{\mathcal{S}}{r}\right) = 1 - \frac{r\mu h}{d} \quad \dots(18)$$

substitute the expression (17) in the expression (18) which provides

$$G\left(\frac{\mathcal{S}}{r}\right) = 1 - \frac{hr}{d} \left\{ \frac{1}{\theta} + e^{-\theta\tau} \left(\frac{k}{2} - \frac{1}{\theta} \right) \right\} \quad \dots(19)$$

Consider the breakdown duration τ follows some specific distribution.

Case (1): Before Truncation Point.

The pdf of τ , $g(\tau)$, is proportional to τ and it is stated as

$$g(\tau) = \begin{cases} b\tau, & 0 \leq \tau \leq \sqrt{2}/b \\ 0, & \text{otherwise} \end{cases}$$

Using the equation (19) in the pdf and get the required optimum reserve inventory as

$$\mathcal{S} = r \sqrt{\frac{2}{b} \left[1 - \frac{hr}{d} \left\{ \frac{1}{\theta} + e^{\theta\tau} \left(\frac{k}{2} - \frac{1}{\theta} \right) \right\} \right]} \quad \dots(20)$$

Case (2): After Truncation Point.

The pdf of τ , $g(\tau)$ follows uniform distribution $u(0,b)$. Using the relation (19), we have $G\left(\frac{\mathcal{S}}{r}\right) = P\left(\tau \leq \frac{\mathcal{S}}{r}\right)$

$$\Rightarrow \mathcal{S} = rb \left[1 - \frac{hr}{d} \left\{ \frac{1}{\theta} + e^{-\theta\tau} \left(\frac{k}{2} - \frac{1}{\theta} \right) \right\} \right] \quad \dots(21)$$

Numerical Illustration:

The numerical values of any such mathematical results show the nature and behaviour of the derived expressions. The purpose of this section is to estimate the optimal value of inventory level, S.

Fix all the parameters and make variations in either holding cost (h) or shortage cost (d). The optimal value of S for the given models are computed. Consider the parameters as fixed and obtain the numerical results for different cases. The fixed numerical values for the parameters are given as

$$\theta_1 = 1, \theta_2 = \theta = 1.5, \lambda = 1, r = 0.5$$

$$\mu = 0.25, t = 5, \tau = 5, b = 1.5, k = 1.5$$

$$d = 30, 35, \dots, 60 \text{ and } h = 20, 25, \dots, 50$$

Case(1)

Table (1): Optimal reserve inventory before truncation point with fixed shortage cost

\mathcal{S} \ h	20	25	30	35	40	45	50
$\widehat{\mathcal{S}}_1$	0.2833	0.2292	0.1924	0.1658	0.1457	0.1299	0.1172
$\widehat{\mathcal{S}}_2$	0.5836	0.5109	0.4516	0.4013	0.3578	0.3199	0.2852
$\widehat{\mathcal{S}}_3$	0.5443	0.4907	0.4714	0.4513	0.4303	0.4083	0.3849

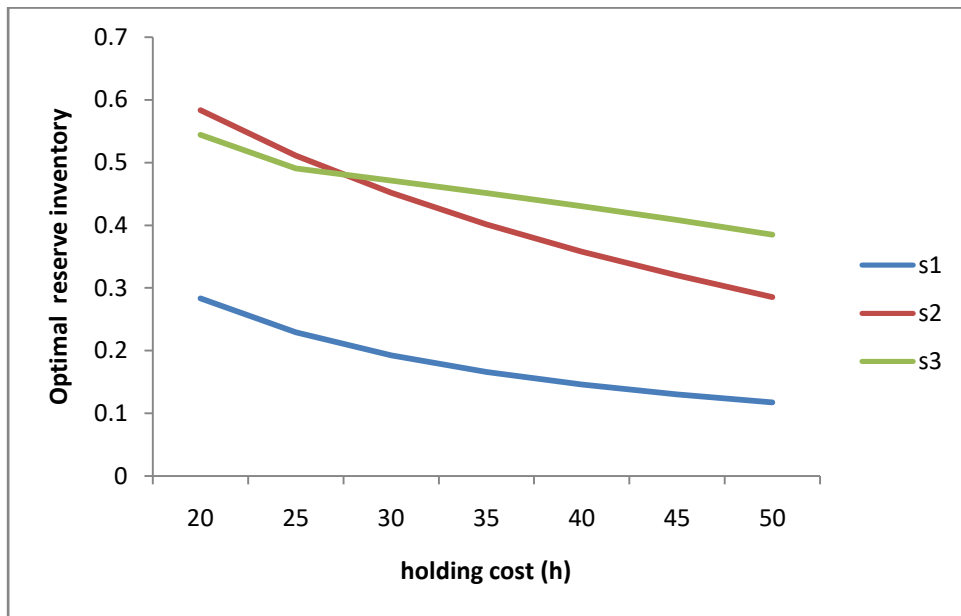


Fig. 1

Table (2): Optimal reserve inventory before the truncation point with fixed holding cost

\mathcal{S} \ d	30	35	40	45	50	55	60
$\widehat{\mathcal{S}}_1$	0.2833	0.3276	0.3711	0.4138	0.4558	0.4971	0.5378
$\widehat{\mathcal{S}}_2$	0.5836	0.6338	0.6773	0.7157	0.7500	0.7810	0.8094
$\widehat{\mathcal{S}}_3$	0.5092	0.5194	0.5270	0.5328	0.5375	0.5417	0.5443

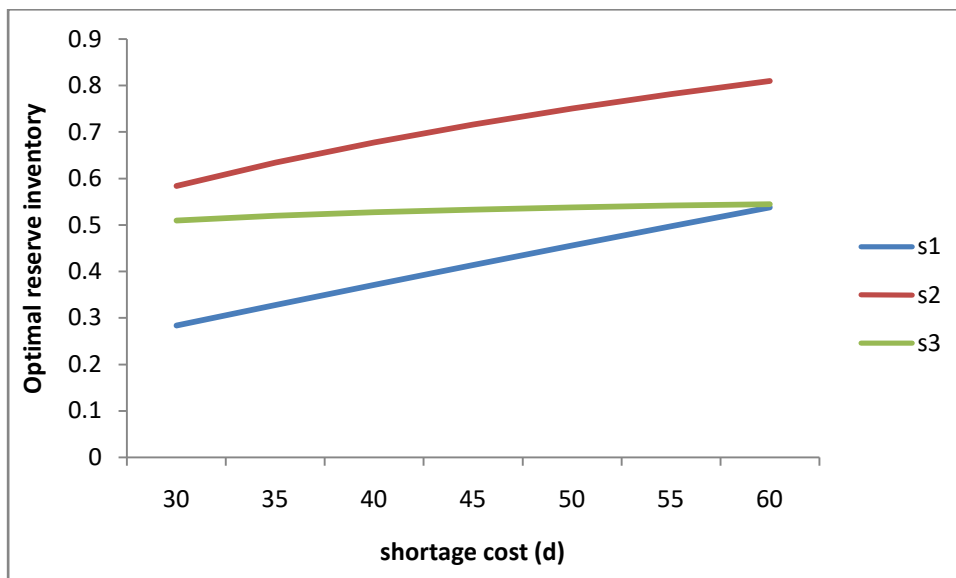


Fig. 2

Case(2)

Table (3): Optimal reserve inventory after the truncation point with fixed shortage cost

\hat{S} \ h	20	25	30	35	40	45	50
\hat{S}_1	3.4524	3.1538	2.9306	2.7553	2.6128	2.4939	2.3925
\hat{S}_2	1.2216	1.1731	1.1336	1.1000	1.0710	1.0458	1.0226
\hat{S}_3	0.5833	0.5417	0.5000	0.4584	0.4167	0.3750	0.3334

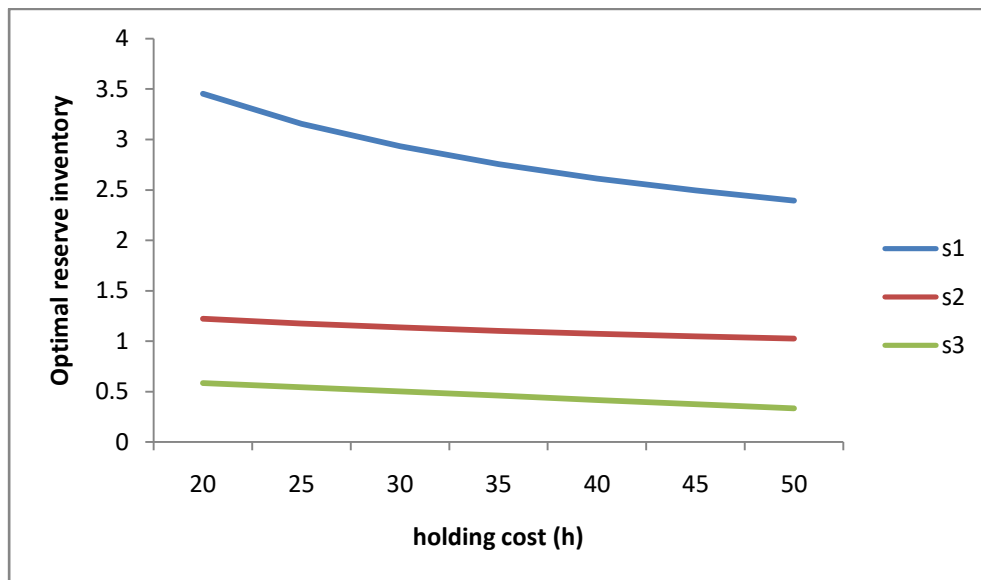


Fig.3

Table (4): Optimal reserve inventory after the truncation point with fixed holding cost

\hat{S} \ d	30	35	40	45	50	55	60
\hat{S}_1	3.4524	3.6764	3.8829	4.0753	4.2557	4.4261	4.4876
\hat{S}_2	1.2216	1.2550	1.2840	1.3096	1.3325	1.3532	1.3721
\hat{S}_3	0.5833	0.6071	0.6250	0.6389	0.6500	0.6591	0.6667

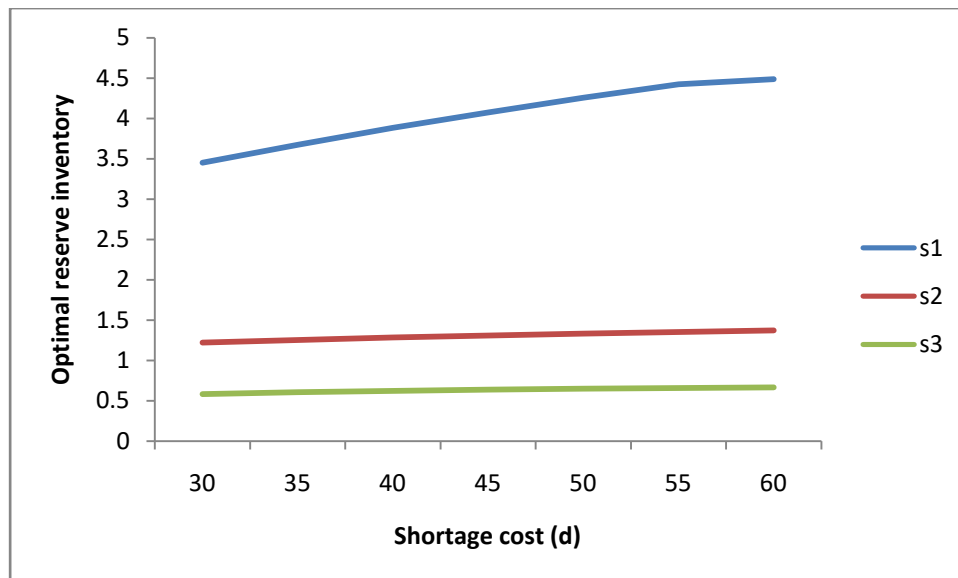


Fig.4

Conclusion

It is attempted to discuss different inventory models. The repair times of the first machine of each model play a vital role to study the levels of reserve inventory. The distributions of the repair times which fall before and after truncation point relating to

each model are differently considered. The optimal reserve inventory for the three models are numerically computed and exhibited.

The figures (1) to (4) revealed the nature of the inventory levels. The breakdown times or repair times of machine M_1 are considered before and after the truncation point. For the repair times before truncation point, the inventory level decreases for increasing holding cost but it increases when the shortage cost increases relating to the three models. It is also noted that the inventory level of the model studied by Sachithanantham et al. (2006) is higher than that of others for fixed cases where the holding cost varies but shortage cost is fixed and vice versa. On the other hand, the model given by Srinivasan et al (2012) provides minimum inventory value for varying holding cost with fixed shortage cost and vice versa. The sequence of inventory levels of the three models is given by $\widehat{\mathcal{S}}_2 > \widehat{\mathcal{S}}_3 > \widehat{\mathcal{S}}_1$.

When the repair time falls after truncation point, the inventory level decreases for increasing holding cost with fixed shortage cost. But the inventory level increases when the shortage cost increases with fixed holding cost. The inventory levels of three models satisfy the inequality $\widehat{\mathcal{S}}_1 > \widehat{\mathcal{S}}_2 > \widehat{\mathcal{S}}_3$.

It is remarked that the inventory level of the model discussed by Srinivasan et al (2012) is minimum when the repair time falls before truncation point but it is maximum after truncation point. The inventory level of the model studied by Nandakumar and Srinivasan (2015) is minimum when the repair time falls after the truncation point.

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